CEN/CLC/JTC 22/WG 3 "Quantum Computing and Simulation"

Convenor: **PAUL Alexandra MME**

HAL_GateDefinitions

Document type	Related content	Document date	Expected action
Meeting / Document for discussion	Meeting: Delft (Netherlands) 29 Apr 2026	2026-04-23	

Description

Dear members,

Please find attached Tables with pre-defined gates for the WI "HAL".

Kind regards,

Simon Del Nin

Tables with pre-defined gates for in the HAL

Date of submission:	2026-04-22
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Expected action:	Vote
Expected date:	2026-04-29 (JTC22/WG3 meeting)
WG3-Project:	HAL

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1. Abstract

Document N215 gives the latest Draft (V02) for a Technical Specification (TS) about the Hardware Abstraction Layer. Chapter 6 is dedicated to instructions, and an important group of instructions is related to gate operations on qubits

This contribution proposes literal text for inclusion into section 6.2, which is dedicated to lists with pre-defined gates.

2. Literal text proposal

Start of literal text proposal

6. Quantum Instructions and Gates

6.1 Instructions

6.2 Quantum gates

Quantum gates are operators that operate one or more qubits to change the overall quantum state. These operations preserve the total probability of quantum states (the norm of the state vector remains 1) and can be described via matrix operations. These matrices **Q** are always unitary, and have the property that the modulus of their determinant |det(**Q**)|=1.
Quantum gates acting on *n* qubits require a multiplication with $2^n \times 2^n$ unitary matrices. Both (classical) matrix multiplications (division) and Kronecker products of unitary matrices result in unitary matrices, so the norm of the state vector is preserved.

A convenient way to define the unitary matrix **Q** associated with a gate in a consistent manner is by means of matrix functions like *expm*(**Q**), *logm*(**Q**) and *sqrtm*(**Q**). and other matrix operations.
Via standard eigenvalue decompositions, these **Q**-matrices can always be decomposed as **Q**=**V**×diag(**d**)/**V**, where **Q** and **V** are square matrices and **d** a column vector. This

property makes the evaluation of matrix functions like *expm*(**Q**), *logm*(**Q**) and *sqrtm*(**Q**) relatively simple. The used functions and symbols are defined as:

- matrix exponent: *expm*(**Q**) = *expm*(**V**×*diag*(**d**)/**V**) = **V**×*diag*(*exp*(**d**))/**V**
- matrix logarithm: *logm*(**Q**) = *logm*(**V**×*diag*(**d**)/**V**) = **V**×*diag*(*log*(**d**))/**V**
- matrix square root: *sqrtm*(**Q**) = *sqrtm*(**V**×*diag*(**d**)/**V**) = **V**×*diag*(*sqrt*(**d**))/**V**
- Kronecker product: $\mathbf{X} \otimes \mathbf{Y}$
- matrix product: $\mathbf{X} \times \mathbf{Y}$
- matrix division: $\mathbf{X}/\mathbf{Y}$ (*right hand division*), $\mathbf{X}\backslash\mathbf{Y}$ (*left hand division*)
- imaginary unit: $j = \sqrt{-1}$

Note that the symbols *i* and *j* refer to the same imaginary unit but *i* and 1 are sometimes too look-alike in expressions that it can be confusing. Therefore *j* is used within this document.

6.2.1 Naming convention for gates within this document

The HAL supports a collection of predefined gates, which can be invoked via a unique name in a gate instruction. Some of these gates are directly provided by the control software layer, while others are emulated within the HAL in order to harmonize gate-sets among different quantum computer implementations.

Some de-facto consensus has been achieved on names and definitions of commonly used gates. This holds for well known gates like X, Y, Z, cNOT, cZ, SWAP, etc, but not for all gates being defined within different implementations. Sometimes the same name is used for different gates, and in other cases different names are used for identical gates. It may be obvious that this can be confusing and error-prone.

In order to harmonize naming and definitions among different HAL implementations, names and definitions of several pre-defined gates are provided in succeeding sections. There is no need that the HAL should support all these gates, nor should it be restricted to these gates. Therefore a HAL implementation should support a mechanism to let higher layers query the names of gates that are pre-defined and how they are defined.

The acceptance of strings with names of predefined gates is case-insensitive. This means that the HAL treats strings like 'cNOT' and 'CNOT' and 'cnot' allways as names for the same predefined gate. The mixed use of lower and upper case in these name is only a matter of style.
The following style convention has been used in this document:

- The base-line name is the fragment of a full name that starts with uppercase characters, and may be followed by a suffix with lowercase characters. For instance base-line names like X, NOT, and SWAP. A suffix with lowercase characters is used when the same expression is used for defining different gates, but differ only on the arguments they apply on. So Rx, Rzx, Rz are examples of such base-line names. Sdg and Tdg is an exception due to historical reasons.
- The full name of an identifier is a base-line name that can be prefixed by lowercase characters. Such a prefix is only used when the gate is derived from another gate by means of a generic matrix function that can operate on any unitary matrix. So "rQ" for *sqrtm*(**Q**) and "cQ" for *controlled*(**Q**). As such, the prefix characters 'r' and 'c' are acting like a modifier from a base-line gate.
- Sometimes a gate has a similarity with another gate but cannot be derived from it via a generic matrix function. In that case it is not a modifier and is written with uppercase characters. For instance SWAP and ISWAP.

The tables in following sections define gates and their associated names, definitions, matrices and symbols.

6.2.2 Single qubit gates

Table 6-1 defines a list with names and definition of various pre-defined gates that operate on a single qubit.

Name	Definition	Matrix	Symbol	Properties
I	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$			$= -\mathbf{X} \times \mathbf{Y} \times \mathbf{Z}$
X	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$		X	$= j \cdot \mathbf{Rx}(\pi)$
Y	$\begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix}$		Y	$= j \cdot \mathbf{Ry}(\pi)$
Z	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$		Z	$= j \cdot \mathbf{Rz}(\pi)$
Rx(a)	$= expm(-j \cdot a/2 \cdot \mathbf{X})$	$\begin{bmatrix} cos(a/2) & -j \cdot sin(a/2) \\ -j \cdot sin(a/2) & cos(a/2) \end{bmatrix}$	Rx(a)	
Ry(a)	$= expm(-j \cdot a/2 \cdot \mathbf{Y})$	$\begin{bmatrix} cos(a/2) & -sin(a/2) \\ sin(a/2) & cos(a/2) \end{bmatrix}$	Ry(a)	
Rz(a)	$= expm(-j \cdot a/2 \cdot \mathbf{Z})$	$\begin{bmatrix} exp(-j \cdot a/2) & 0 \\ 0 & exp(j \cdot a/2) \end{bmatrix}$	Rz(a)	
P(a)	$= expm(-j \cdot a/2 \cdot (\mathbf{Z} - \mathbf{I}))$	$\begin{bmatrix} 1 & 0 \\ 0 & exp(j \cdot a) \end{bmatrix}$	P(a)	
S	$= expm(-j\pi/4 \cdot (\mathbf{Z} - \mathbf{I}))$	$\begin{bmatrix} 1 & 0 \\ 0 & j \end{bmatrix}$	S	$= \mathbf{P}(\pi/2)$ $= sqrtm(\mathbf{Z})$
T	$= expm(-j\pi/8 \cdot (\mathbf{Z} - \mathbf{I}))$	$\begin{bmatrix} 1 & 0 \\ 0 & (1+j)/\sqrt{2} \end{bmatrix}$	T	$= \mathbf{P}(\pi/4)$ $= sqrtm(\mathbf{S})$
Sdg	$= expm(+j\pi/4 \cdot (\mathbf{Z} - \mathbf{I}))$	$\begin{bmatrix} 1 & 0 \\ 0 & -j \end{bmatrix}$	$S^\dagger$	$= \mathbf{P}(-\pi/2)$
Tdg	$= expm(+j\pi/8 \cdot (\mathbf{Z} - \mathbf{I}))$	$\begin{bmatrix} 1 & 0 \\ 0 & (1-j)/\sqrt{2} \end{bmatrix}$	$T^\dagger$	$= \mathbf{P}(-\pi/4)$
V	$= expm(-j\pi/4 \cdot (\mathbf{X} - \mathbf{I}))$	$\begin{bmatrix} 1+j & 1-j \\ 1-j & 1+j \end{bmatrix}/2$	V	$= sqrt(j) \cdot \mathbf{Rx}(\pi/2)$ $= sqrtm(\mathbf{X})$
H	$= j \cdot \mathbf{Rx}(\pi) \times \mathbf{Ry}(\pi/2)$	$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}/\sqrt{2}$	H	$= \mathbf{X} \times sqrtm(-j\mathbf{Y})$ $= \mathbf{Z} \times sqrtm(+j\mathbf{Y})$ $= sqrtm(-j\mathbf{Y}) \times \mathbf{Z}$ $= sqrtm(+j\mathbf{Y}) \times \mathbf{X}$
NOT	same as **X**			$= \mathbf{X}$
rNOT	same as **V**			$= sqrtm(\mathbf{NOT})$ $= sqrtm(\mathbf{X})$

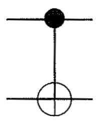
Table 6-1 *List with pre-defined gates operating on a single qubit*

6.2.3 Double qubit gates

Table 6-2 to 6-4 define lists with names and definition of various pre-defined gates that operate on two qubits.

Name	Definition	Matrix	Properties
Rxx(a)	$= expm(-j \cdot a/2 \cdot \mathbf{X} \otimes \mathbf{X})$	$\begin{bmatrix} cos(a/2) & 0 & 0 & -j \cdot sin(a/2) \\ 0 & cos(a/2) & -j \cdot sin(a/2) & 0 \\ 0 & -j \cdot sin(a/2) & cos(a/2) & 0 \\ -j \cdot sin(a/2) & 0 & 0 & cos(a/2) \end{bmatrix}$	$\mathbf{X} \otimes \mathbf{X} = j \cdot \mathbf{Rxx}(\pi)$
Ryy(a)	$= expm(-j \cdot a/2 \cdot \mathbf{Y} \otimes \mathbf{Y})$	$\begin{bmatrix} cos(a/2) & 0 & 0 & j \cdot sin(a/2) \\ 0 & cos(a/2) & -j \cdot sin(a/2) & 0 \\ 0 & -j \cdot sin(a/2) & cos(a/2) & 0 \\ j \cdot sin(a/2) & 0 & 0 & cos(a/2) \end{bmatrix}$	$\mathbf{Y} \otimes \mathbf{Y} = j \cdot \mathbf{Ryy}(\pi)$
Rzz(a)	$= expm(-j \cdot a/2 \cdot \mathbf{Z} \otimes \mathbf{Z})$	$\begin{bmatrix} exp(-j \cdot a/2) & 0 & 0 & 0 \\ 0 & exp(j \cdot a/2) & 0 & 0 \\ 0 & 0 & exp(j \cdot a/2) & 0 \\ 0 & 0 & 0 & exp(-j \cdot a/2) \end{bmatrix}$	$\mathbf{Z} \otimes \mathbf{Z} = j \cdot \mathbf{Rzz}(\pi)$
Rxy(a)	$= expm(-j \cdot a/2 \cdot \mathbf{X} \otimes \mathbf{Y})$	$\begin{bmatrix} cos(a/2) & 0 & 0 & -sin(a/2) \\ 0 & cos(a/2) & sin(a/2) & 0 \\ 0 & -sin(a/2) & cos(a/2) & 0 \\ sin(a/2) & 0 & 0 & cos(a/2) \end{bmatrix}$	$\mathbf{X} \otimes \mathbf{Y} = j \cdot \mathbf{Rxy}(\pi)$
Ryx(a)	$= expm(-j \cdot a/2 \cdot \mathbf{Y} \otimes \mathbf{X})$	$\begin{bmatrix} cos(a/2) & 0 & 0 & -sin(a/2) \\ 0 & cos(a/2) & -sin(a/2) & 0 \\ 0 & sin(a/2) & cos(a/2) & 0 \\ sin(a/2) & 0 & 0 & cos(a/2) \end{bmatrix}$	$\mathbf{Y} \otimes \mathbf{X} = j \cdot \mathbf{Ryx}(\pi)$
Rzx(a)	$= expm(-j \cdot a/2 \cdot \mathbf{Z} \otimes \mathbf{X})$	$\begin{bmatrix} cos(a/2) & -j \cdot sin(a/2) & 0 & 0 \\ -j \cdot sin(a/2) & cos(a/2) & 0 & 0 \\ 0 & 0 & cos(a/2) & j \cdot sin(a/2) \\ 0 & 0 & j \cdot sin(a/2) & cos(a/2) \end{bmatrix}$	$\mathbf{Z} \otimes \mathbf{X} = j \cdot \mathbf{Rzx}(\pi)$
Rzy(a)	$= expm(-j \cdot a/2 \cdot \mathbf{Z} \otimes \mathbf{Y})$	$\begin{bmatrix} cos(a/2) & -sin(a/2) & 0 & 0 \\ sin(a/2) & cos(a/2) & 0 & 0 \\ 0 & 0 & cos(a/2) & sin(a/2) \\ 0 & 0 & -sin(a/2) & cos(a/2) \end{bmatrix}$	$\mathbf{Z} \otimes \mathbf{Y} = j \cdot \mathbf{Rzy}(\pi)$
Rxz(a)	$= expm(-j \cdot a/2 \cdot \mathbf{X} \otimes \mathbf{Z})$	$\begin{bmatrix} cos(a/2) & 0 & -j \cdot sin(a/2) & 0 \\ 0 & cos(a/2) & 0 & j \cdot sin(a/2) \\ -j \cdot sin(a/2) & 0 & cos(a/2) & 0 \\ 0 & j \cdot sin(a/2) & 0 & cos(a/2) \end{bmatrix}$	$\mathbf{X} \otimes \mathbf{Z} = j \cdot \mathbf{Rxz}(\pi)$
Ryz(a)	$= expm(-j \cdot a/2 \cdot \mathbf{Y} \otimes \mathbf{Z})$	$\begin{bmatrix} cos(a/2) & 0 & -sin(a/2) & 0 \\ 0 & cos(a/2) & 0 & sin(a/2) \\ sin(a/2) & 0 & cos(a/2) & 0 \\ 0 & -sin(a/2) & 0 & cos(a/2) \end{bmatrix}$	$\mathbf{Y} \otimes \mathbf{Z} = j \cdot \mathbf{Ryz}(\pi)$

Table 6-2 *List with pre-defined two-qubit rotation gates*

name	definition	matrix	symbols
		$\mathbf{C} \equiv \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$	**C** is not a gate but an auxiliary matrix
cNOT	same as **cX**	same as **cX**	

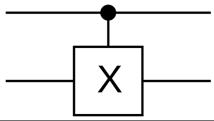
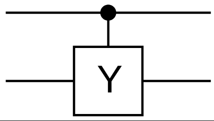
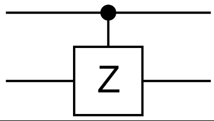
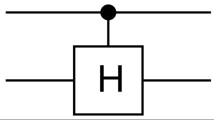
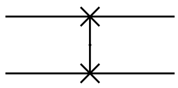
cX	$= expm(\mathbf{C} \otimes logm(\mathbf{X}))$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	
cY	$= expm(\mathbf{C} \otimes logm(\mathbf{Y}))$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -j \\ 0 & 0 & j & 0 \end{bmatrix}$	
cZ	$= expm(\mathbf{C} \otimes logm(\mathbf{Z}))$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$	
cS	$= expm(\mathbf{C} \otimes logm(\mathbf{S}))$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & j \end{bmatrix}$	
cSdg	$= expm(\mathbf{C} \otimes logm(\mathbf{Sdg}))$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -j \end{bmatrix}$	
cT	$= expm(\mathbf{C} \otimes logm(\mathbf{T}))$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & (1+j)/\sqrt{2} \end{bmatrix}$	
cTdg	$= expm(\mathbf{C} \otimes logm(\mathbf{Tdg}))$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & (1-j)/\sqrt{2} \end{bmatrix}$	
cH	$= expm(\mathbf{C} \otimes logm(\mathbf{H}))$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{1/2} & \sqrt{1/2} \\ 0 & 0 & \sqrt{1/2} & -\sqrt{1/2} \end{bmatrix}$	
cP	$= expm(\mathbf{C} \otimes logm(\mathbf{P}))$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & exp(j \cdot a) \end{bmatrix}$	

***Table 6-3** List with pre-defined controlled gates operating on two qubits*

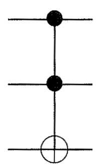
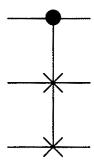
Name	definition + matrix	Properties + Symbol
SWAP	$= expm(-j\pi/4 \cdot (\mathbf{X} \otimes \mathbf{X} + \mathbf{Y} \otimes \mathbf{Y} + \mathbf{Z} \otimes \mathbf{Z} - \mathbf{I} \otimes \mathbf{I}))$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	
rSWAP	$= expm(-j\pi/8 \cdot (\mathbf{X} \otimes \mathbf{X} + \mathbf{Y} \otimes \mathbf{Y} + \mathbf{Z} \otimes \mathbf{Z} - \mathbf{I} \otimes \mathbf{I}))$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & (1+j)/2 & (1-j)/2 & 0 \\ 0 & (1-j)/2 & (1+j)/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$= sqrtm(\mathbf{SWAP})$
ISWAP	$= expm(-j\pi/4 \cdot (\mathbf{X} \otimes \mathbf{X} + \mathbf{Y} \otimes \mathbf{Y}))$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & j & 0 \\ 0 & j & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	

rISWAP	$= expm(-j\pi/8 \cdot (\mathbf{X} \otimes \mathbf{X} + \mathbf{Y} \otimes \mathbf{Y}))$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & j/\sqrt{2} & 0 \\ 0 & j/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$= sqrtm(\mathbf{ISWAP})$

Table 6-4 *List with various other pre-defined gates operating on two qubits*

6.2.4 Multiple qubit gates

Table 6-5 defines a list with names and definition of various pre-defined gates that operate on more than two qubits. The systematic used within the expressions used for defining them illustrate how to extend this list in case a gate is needed with multiple controls.

Name	definition + matrix	Properties + Symbol
	$\mathbf{C} \equiv \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$	**C** is not a gate but an auxiliary matrix
ccNOT	same as **ccX**	
ccX	$= expm(\mathbf{C} \otimes \mathbf{C} \otimes logm(\mathbf{X}))$ (*also known as Toffoli gate*) $\begin{bmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&1&0 \end{bmatrix}$	
cSWAP	$= expm(\mathbf{C} \otimes logm(\mathbf{SWAP}))$, (*also known as the Fredkin gate*) $\begin{bmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1 \end{bmatrix}$	
crSWAP **rcSWAP**	$= expm(\mathbf{C} \otimes logm(\mathbf{rSWAP}))$ $= expm(\mathbf{C} \otimes logm(\mathbf{SWAP})/2)$ $\begin{bmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&(1+j)/2&(1-j)/2&0 \\ 0&0&0&0&0&(1-j)/2&(1+j)/2&0 \\ 0&0&0&0&0&0&0&1 \end{bmatrix}$	crSWAP and rcSWAP give the same results $= sqrtm(\mathbf{cSWAP})$
cISWAP	$= expm(\mathbf{C} \otimes logm(\mathbf{iSWAP}))$ $\begin{bmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&j&0 \\ 0&0&0&0&0&j&0&0 \\ 0&0&0&0&0&0&0&1 \end{bmatrix}$	

crISWAP **rcISWAP**	$= expm(\mathbf{C} \otimes logm(\mathbf{riSWAP}))$ $= expm(\mathbf{C} \otimes logm(\mathbf{iSWAP})/2)$ $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/\sqrt{2} & j/\sqrt{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & j/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	crISWAP and rcISWAP give the same results $= sqrtm(\mathbf{cISWAP})$

Table 6-5 *List with various other pre-defined gates operating on three qubits*

End of literal text proposal